

CLASS-XII <u>RELATIONS AND FUNCTIONS</u> <u>KEY POINTS</u> <u>TYPES OF RELATIONS</u>	
<u>MULTIPLE CHOICE QUESTIONS</u>	
Qn.1	Let R be a relation on the set N be defined by $R = \{(x, y) \mid \forall x, y \in N, 2x + y = 41\}$. Then, R is (a) Reflexive (b) Symmetric (c) Transitive (d) None of these
Qn.2	For real numbers x and y, we write $x R y \leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is (a) Reflexive (b) Symmetric (c) Transitive (d) None of these
Qn.3	The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is (a) Reflexive but not symmetric (b) Reflexive but not transitive (c) Symmetric and transitive (d) Neither symmetric nor transitive
Qn. 4	Consider the non-empty set consisting of children in a family and a relation R defined as $a R b$ if a is brother of b. Then R is (a) symmetric but not transitive (b) transitive but not symmetric (c) neither symmetric nor transitive (d) both symmetric and transitive
Qn. 5	Let $P = \{(x, y) : x^2 + y^2 = 1, x, y \in \mathbb{R}\}$. Then, P is (a) Reflexive (b) Symmetric (c) Transitive (d) Anti-symmetric
Qn.6	Let S be the set of all real numbers. Then, the relation $R = \{(a, b) : 1 + ab > 0\}$ on S is (a) Reflexive and symmetric but not transitive (b) Reflexive and transitive but not symmetric (c) Symmetric, transitive but not reflexive (d) Reflexive, transitive and symmetric

Qn.7	Let R be the relation in the set Z of all integers defined by $R = \{(x, y) : x - y \text{ is an integer}\}$. Then R is (a) reflexive (b) symmetric (c) transitive (d) an equivalence relation
Qn.8	For the set $A = \{1, 2, 3\}$, define a relation R in the set A as follows $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ Then, the ordered pair to be added to R to make it the smallest equivalence relation is (a) (1, 3) (b) (3, 1) (c) (2, 1) (d) (1, 2)
Qn.9	Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 3)\}$ be a relation in A. Then, the minimum number of ordered pairs may be added, so that R becomes an equivalence relation, is (a) 7 (b) 5 (c) 1 (d) 4
Qn.10	Let $A = \{1, 2, 3\}$. Then, the number of relations containing (1, 2) and (1, 3), which are reflexive and symmetric but not transitive, is (a) 1 (b) 2 (c) 3 (d) 4
Qn.11	Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^3 + 4$, then f is (a) Injective (b) Surjective (c) Bijective (d) None of these
Qn.12	Let $X = \{0, 1, 2, 3\}$ and $Y = \{-1, 0, 1, 4, 9\}$ and a function $f : X \rightarrow Y$ defined by $y = x^2$, is (a) one-one onto (b) one-one into (c) many-one onto (d) many-one into
Qn.13	Let $g : \mathbb{R} \rightarrow \mathbb{R}$ $g(x) = x^2 - 4x - 5$, then (a) g is one-one on R (b) g is not one-one on R (c) g is bijective on R (d) None of these
Qn.14	The mapping $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = 1 + n^2$, $n \in \mathbb{N}$ when N is the set of natural numbers, is (a) a one-one function (b) an onto function (c) a bijection (d) neither one-one nor onto
Qn.15	The function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^3 - 1$ is (a) a one-one function (b) an onto function (c) a bijection (d) neither one-one nor onto

Qn.16	A function $f : X \rightarrow Y$ is said to be onto, if for every $y \in Y$, there exists an element x in X such that (a) $f(x) = y$ (b) $f(y) = x$ (c) $f(x) + y = 0$ (d) $f(y) + x = 0$
Qn.17	Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. (a) R is reflexive and symmetric but not transitive (b) R is reflexive and transitive but not symmetric (c) R is symmetric and transitive but not (d) R is equivalence relation
Qn.18	Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, then the number of bijective functions from A to B are (a) 2 (b) 8 (c) 6 (d) 4
Qn.19	The number of surjective functions from A to B where $A = \{1, 2, 3, 4\}$ and $B = \{a, b\}$ is (a) 14 (b) 12 (c) 2 (d) 15
Qn.20	The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is (a) one-one but not onto (b) onto but not one-one (c) both one-one and onto (d) neither one-one nor onto
<u>OBJECTIVE QUESTIONS</u>	
Qn.1	If $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ Then, the ordered pair to be added to R to make it the smallest equivalence relation is
Qn.2	Check " xy is square of an integer $x, y \in \mathbb{N}$ ". Above relation is (Reflexive, symmetric and transitive.)
Qn.3	Give an example of a function which is one-one but not onto.....
Qn.4	Let D be the domain of the real valued function f defined by $f(x) = \sqrt{25 - x^2}$. Then, write D
Qn.5	If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ a function? If g is described by $g(x) = ax + b$, then value of a is.....and value of b is.....
Qn.6	Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$: be defined by $f(x) = \cos x; \forall x \in \mathbb{R}$. Show that f is neither one-one nor onto.
Qn.7-10	MATCH THE FOLLOWING

	7. $f: \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^2$ 8. $f: \mathbb{R} \rightarrow \mathbb{N} : f(x) = x^2$ 9. $f: \mathbb{Z} \rightarrow \mathbb{Z} : f(x) = x^3$ 10. $f: \mathbb{N} \rightarrow \mathbb{N} : f(x) = x^2$	(i) Neither one one nor onto (ii) One one and into (iii) Many one and onto (iv) One one but not onto
Qn.11	Examine if the relation $g = \{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$ is one one onto State the reason.	
Qn.12	If $R = \{(x, y) : x + 2y = 8\}$ is a relation in \mathbb{N} , write the Domain of R .	
Qn.13	If a function $f: X \rightarrow (-1,1)$ defined by $y = \cos x$ is one-one and onto function, then value of $X = \dots\dots\dots$	
Qn.14	Let A be the set containing 'm' distinct elements, then the total number of distinct functions from A to itself is : $\dots\dots\dots$	
Qn.15	If $A = \{1,2,3,4\}$ and $B = \{1,2,3\}$, then number of mappings from A to B is $\dots\dots$	
Qn.16	If $f: [0, \frac{\pi}{2}] \rightarrow [0, \infty]$ be a function defined by $y = \sin(\frac{x}{2})$, then f is $\dots\dots\dots$ (injective/surjective/bijective)	
Qn.17	If a function $f: [2, \infty) \rightarrow B$ defined by $f(x) = x^2 - 4x + 5$ is a bijection, then $B = \dots\dots\dots$	
Qn.18	Let $R = \{(a, a^3) : a \text{ is a prime number less than } 4\}$ be a relation. Then the range of R is $\dots\dots\dots$	
Qn.19	If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is $\dots\dots\dots$ (reflexive/symmetric/transitive)	
Qn.20	Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$, then pre-image of 17 and -3, respectively, are $\dots\dots\dots$	
SHORT ANSWER QUESTIONS		
Qn.1	Find the type of relation, which is describes as the relation "less than" in the set of natural numbers.	
Qn.2	Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = 2x$ is one-one but not onto	
Qn.3	Show that the function: $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$ is onto but not one-one.	
Qn.4	If $R = \{(x, y) : x + 2y = 8\}$ is a relation in \mathbb{N} , write the range of R .	
Qn.5	If the function $f: \mathbb{R} \rightarrow A$ given by $f(x) = \frac{x^2}{x^2 + 1}$ is a surjection, then find A	
Qn.6	If the relation R is defined by aRb , if and only, if b lives within one kilometer from a , then check if the relation is reflexive, symmetric or transitive.	

Qn.7	The function $f: X \rightarrow Y$ defined by $f(x) = \sin x$ is one-one but not onto, then find X and Y.
Qn.8	Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x & \text{if } x > 3 \\ x^2 & \text{if } 1 < x \leq 3 \\ 3x & \text{if } x \leq 1. \end{cases}$ Then find the value of $f(-1) + f(2) + f(4)$
Qn.9	On the set of integers Z , define $f: Z \rightarrow Z$ as $f(n) = \begin{cases} \frac{n}{2} & \text{if } n = \text{odd} \\ 0, & \text{if } n = \text{even} \end{cases}$ then Check whether the function is injective or surjective or none.
Qn.10	Find the maximum number of equivalence relations on the set $A = \{1, 2, 3\}$
Qn.11	If R denotes the set of all real numbers, then the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x $ is neither one- one nor onto. Justify the statement.
Qn.12	Find the range of $f(x) = \frac{x}{x+1}$
Qn.13	If a function f is defined as $\{(1,1)(2,3)(3,5)(4,7)\}$ is described as $f(x) = ax + b$, find a and b .
Qn.14	Write the smallest equivalence relation of the set $A = \{1, 2, 3\}$
Qn.15	Prove that the relation $= \{a-b = \text{even numbers}\}$ defined on a set of integers is equivalence relation.
Qn.16	Let $A = \{a, b, c\}$ and the relation R be defined on A as follows $R = \{(a, a), (b, c), (a, b)\}$ Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.
Qn.17	Let $A = \{a, b, c\}$ and the relation R be defined on A as follows $R = \{(a, a), (b, c), (a, b)\}$ Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.
Qn.18	If $R = \{(x, y) : x^2 + y^2 \leq 4 ; x, y \in Z\}$ is a relation on Z , find the range of R .
Qn.19	Give an example of a relation which is symmetric and transitive but not reflexive.
Qn.20	Show that $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n = \text{odd} \\ \frac{n}{2}, & \text{if } n = \text{even} \end{cases}$ is many onto function
CASE STUDY QUESTIONS	

Qn.1	<p>Aman and Ramesh are playing Ludo at home during Covid-19. While rolling the dice, Aman's sister Lata observed and noted the possible outcomes of the throw every time belongs to set $\{1,2,3,4,5,6\}$. Let A be the set of players while B be the set of all possible outcomes. Let $A=\{A,R\}, B=\{1,2,3,4,5,6\}$. Using the information given above, answer the following:</p> <p>(i) Let $R:B\rightarrow B$ be defined by $R = \{(x,y) : y = x\}$ is (a) Reflexive and transitive but not symmetric (b) Reflexive and symmetric but not transitive (c) Reflexive but not symmetric and transitive (d) Equivalence</p> <p>(ii) Let $R :B\rightarrow B$ be defined by $R=\{(1,2)(2,2)(1,3)(3,4)(3,1)(4,3)(5,5)\}$. Then R is (a) Symmetric (b) Reflexive (c) Transitive (d) None of these three</p> <p>(iii) Let $R :B\rightarrow B$ be defined by $R=\{(2,1)(1,2)(2,2)(3,3)(4,4)(5,5)(6,6)\}$, then R is (a) Symmetric (b) Reflexive and Transitive (c) Transitive and symmetric (d) Equivalence</p> <p>(iv) Lata wants to know the number of relations possible from A to B .How many relations are possible? a)36 (b) 64 (c) 6! (d) 2^{12}</p> <p>(v) Lata wants to know the number of functions from $A\rightarrow B$, How many numbers of functions are possible? (a)36 (b) 64 (c) 6! (d) 2^{12}</p>
Qn.2	<p>Ved visited the exhibition along with her family. The exhibition had a huge swing which attracted many children. Ved found that the swing traced the path of a parabola as given by $y=x^2$.Answer the questions, using the above information:</p> <p>(i) Let $f: R\rightarrow R$ be defined by $f(x)= x^2$ is (a) Neither surjective nor injective (b) surjective (c) Injective (d) Bijective</p> <p>(ii) Let $f: N\rightarrow N$ be defined by $f(x)= x^2$ is..... (a) Surjective but not injective (b) Surjective (c) Injective (d) Bijective</p> <p>(iii) Let $f: \{1,2,3,\dots\} \rightarrow \{1,4,9,\dots\}$ be defined by $f(x)= x^2$ (a) Surjective but not injective (b) Neither surjective nor injective (c) Injective but not surjective (d) Bijective</p>

	<p>(iv) Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Then, range of the function among the following is</p> <p>(a) $\{1,4,9,16 \dots\}$ (b) $\{1,4,8,9,10 \dots\}$ (c) $\{1,4,9,15,16, \dots\}$ (d) $\{1,4,8,16, \dots\}$</p> <p>(v) If the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by $f(x) = x^2$ is</p> <p>(a) Neither surjective nor injective (b) surjective (c) Injective (d) Bijective</p>
Qn.3	<p>Students of class 11 th, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted on the rows of the saplings along the line $y=2x+4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L. Answer the following, using the above information:</p> <p>(i) Let R be a relation defined by $R = \{(L_1, L_2) : L_1 \parallel L_2 \text{ where } L_1, L_2 \in L\}$, then R is relation</p> <p>(a) Equivalence (b) only reflexive (c) Not reflexive (d) Symmetric but not transitive</p> <p>(ii) Let R be a relation defined by $R = \{(L_1, L_2) : L_1 \perp L_2 \text{ where } L_1, L_2 \in L\}$, then R is relation</p> <p>(a) Equivalence (b) only reflexive (c) Transitive (d) Symmetric but not transitive and reflexive</p> <p>(b)</p> <p>(iii) Let function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x+4$</p> <p>(a) Surjective but not injective (b) Neither surjective nor injective (c) Injective but not surjective (d) Bijective</p> <p>(iv) The family of lines which are parallel to $f(x) = 2x+4$ is represented by</p> <p>(a) $y = x+c$ (b) $y = 2x+k$ (c) $y = -2x+k$ (d) $y = -x+k$</p> <p>(v) Let function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x+4$, then range of $f(x) =$</p> <p>(a) \mathbb{R} (b) \mathbb{Z} (c) \mathbb{W} (d) \mathbb{Q}</p>
Qn.4	<p>A Robot works on the software which follows function $f(x) = \frac{x-2}{x-1}$. If the value of domain is put in place of x. This robot works and performs various works. Based on the above information, answer the following:</p> <p>(i) What will the value/values of x, on which this robot works</p> <p>(a) On all real values (b) On all real values except 1 (c) On all real values except 2 (d) On all real values except $\{1,2\}$</p> <p>(ii) If range denotes the number of works performed, then range of the works performed will be</p>

	<p>(a) $R - \{1\}$ (b) $R - \{2\}$ (c) $R - \{1,2\}$ (d) On all real values</p> <p>(iii) If this function is defined from $f:R-\{1\} \rightarrow R - \{1\}$ (a)Injective (b) Surjective (c)Bijjective (d) Into</p> <p>(iv) If a Robot follows the $f:R-\{1\} \rightarrow R$, then $f(x)$ is (a)Injective (b) Surjective (c)Bijjective (d) Into</p> <p>(v) If a Robot follows the $f:N-\{1\} \rightarrow R - \{1\}$, then $f(x)$ is (a)Injective (b) Surjective (c)Bijjective (d) Into</p>
Qn.5	<p>An organization conducted bike race under 2 different categories – boys and girls. Totally there were 250 participants. Among all of them finally three from category -1 and two from category-2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B=\{b_1, b_2, b_3\}$ $G=\{g_1, g_2\}$, where B represents the set of boys and G the set of girls who were selected for the final race. Ravi decides to explore these sets for various types of relations and functions.</p> <p>(i). Ravi wishes to form all the relations possible from B to G .How many such relations are possible? (a)2^6 (b) 2^5 (c) 2^3 (d) 0</p> <p>(ii). Let $R:B \rightarrow B$ be defined by $R=\{(x,y): x \text{ and } y \text{ are students of same sex}\}$, Then this relation R is (a)Equivalence (b) Reflexive (c) Reflexive and symmetric but not transitive (d) Reflexive an transitive but not symmetric.</p> <p>(iii). Ravi wants to know among those relations, how many functions can be formed from B to G? (a)2^2 (b) 2^{12} (c) 3^2 (d) 2^3</p> <p>(iv). Let $R: B \rightarrow G$ be defined by $R=\{(b_1, g_1)(b_2, g_2)(b_3, g_1)\}$, then R is..... (a)Injective (b)Surjective (c)Neither Surjective nor Injective (d)Surjective and Injective</p>

	<p>(v)Ravi wants to find the number of injective functions from B to G .How many numbers of injective functions are possible. (a)0 (b)2! (c)3! (d)0!</p>
LONG ANSWER QUESTIONS	
Qn.1	Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. If $f: A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}$, $\forall x \in A$. Then, show that f is bijective
Qn.2	If $A = \{1, 2, 3, 4\}$ define relations on A which have properties of being (i) reflexive, transitive but not symmetric. (ii) Symmetric but neither reflexive nor transitive. (iii) Reflexive, symmetric and transitive
Qn.3	Let $A = \{1, 2, 3, 4, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalent class. $[(2, 5)]$.
Qn.4	Let $A = \mathbb{N} \times \mathbb{N}$ be the set of all ordered pairs of natural numbers and R be the relation on the set A defined by $(a, b) R (c, d)$ iff $ad = bc$. Show that R is an equivalence relation
Qn.5	Show that the relation R on \mathbb{R} defined as $R = \{(a, b): a \leq b\}$, is reflexive and transitive but not symmetric.
Qn.6	Let $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A; a - b \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class $[2]$.
Qn.7	Check whether the relation R in the set \mathbb{R} of real numbers, defined by $R = \{(a, b): 1 + ab > 0\}$, is reflexive, symmetric or transitive.
Qn.8	Let \mathbb{N} denote the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b) R (c, d)$ is $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation
Qn.9	Let $f: \mathbb{R}^+ \rightarrow [-9, \infty)$ be a function defined as $f(x) = 5x^2 + 6x - 9$. Show that $f(x)$ is bijective
Qn.10	Let $f: [-1, \infty) \rightarrow [-1, \infty)$ is given by $f(x) = (x^2 + 1)^2 - 1$, $x \geq 1$. Show that f is bijective